Notre Dame Journal of Formal Logic
Volume 23, Number 1, January 1982

Model Structures and Set Algebras for Sugihara Matrices

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1 Introduction Since the work of Lemmon on modal algebras [2], [3], it has been known that there is a close relationship between relational model structures, set algebras, and matrices. The type of result which Lemmon established was to show how to construct a modal set algebra given a modal model structure, or a model structure given an algebra, in such a way that validity in the algebra coincides with validity in the model structure.

The extension of Lemmon's type of result to various cases of relevant algebras and relevant model structures associated with relevant logics has been studied by Brady in [6], by Routley and Meyer in [5] and [6], and by the author in [4] and [6]. The purpose of this paper is to report results connecting model structures and set algebras for the Sugihara matrices, and in particular for two infinite Sugihara matrices, both of which are characteristic for the important logic RM. The Sugihara matrices, or chains, and the logic RM are investigated in Anderson and Belnap's [1]. To date, no semantics for RM has been given which uses only a single relational model structure. This paper provides such a semantics. Earlier results, for example in [6], of such theorems connecting particular relational model structures and particular set algebras have been exclusively for finite cases of such algebras. The present result is new in that it is the first such example of an infinite algebra and model structure.

2 Sugihara matrices, algebras, and model structures Let I be the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, and let $I^+ = I \cup \{+\omega, -\omega\}$. The ordering $<_e$ on I^+, called the *extensional ordering*, is defined to be the natural ordering on I together with the proviso that for $x \in I$, $-\omega < x < +\omega$. The *intensional ordering*, $<_i$, on I^+ is defined by $x <_i y$ if either $|x| <_e |y|$ (where $|-\omega| = +\omega$), or $x = -y$ and $x <_e 0$. Let $s_n^0 = \{x : x \in I \;\&\; |x| <_e n + 1\}$, and let $s_n = s_n^0 - \{0\}$. The *Sugihara matrices* are quintuples $\langle \Sigma, \vee, \sim, \rightarrow, \mathcal{D} \rangle$ where: (a) Σ is I^+, $I^+ - \{0\}$, I, $I - \{0\}$, s_n^0, or s_n; (b) $x \vee y = \max\{x, y\}$ relative to $<_e$; (c) $\sim x$ = the numerical

Received May 2, 1980; revised March 30, 1981