Notre Dame Journal of Formal Logic
Volume 22, Number 3, July 1981

Careful Choices—A Last Word on Borel Selectors

To the memory of C. D. Papakyriakopoulos

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Selector theory as surveyed in [13] and [14] deals with the following problem (instances of which arise in control theory, probability, mathematical economics, operator theory, etc.): We are given a *multifunction* F between reasonable spaces T and X (a map assigning each $t \in T$ a nonempty $F(t) \subseteq X$) and seek an ordinary function f from T to X with acceptable measurability properties constituting a *selector* for F (satisfying $f(t) \in F(t)$ for all t). Of course, the Axiom of Choice says that a selector exists; but to get a measurable one, we need to impose hypotheses on F and choose "carefully". The past few years have seen much progress (cf. [10], [11], [14]) on the Borel case of the selector problem. In this case we assume X is a *Polish* topological space (one admitting a countable basis and a complete metric) and T at least a *Suslin* space (homeomorph of an analytic subspace of a Polish space). Our goal is to find weak hypotheses on F guaranteeing the existence of a *Borel-measurable* selector f (one for which $f^{-1}[U]$ is Borel in T whenever U is open in X).

The present paper* shows that substantial improvements of existing results on the Borel selector problem can be achieved through application of ideas developed by Vaught in his prize-winning studies [12] on the model theory of infinitary logic. The precise statement of the result obtained is given in Section 3 below. Thanks to certain counterexamples, we can say that this result is in many ways "best possible". Selector theory is thus a relatively down-to-earth area of mathematics where methods from modern logical research can be fruitfully applied.

*The author is indebted to Dan Mauldin, Douglas Miller, Shashi Srivastava, and Daniel Wagner for many helpful discussions.

Received January 2, 1980