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Cofinal Extensions of Nonstandard Models of Arithmetic

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A relatively neglected aspect of the study of nonstandard models of arithmetic is the study of their cofinal extensions.* These extensions certainly do not present themselves to the intuition as readily as do their more popular cousins the end extensions; but they are not exactly shrouded in mystery or unnatural objects of study either. They are equal partners with end extensions in the construction of general extensions of models; they offer both special advantages and disadvantages worthy of our interest; and, occasionally, they are useful in understanding the generally more simply behaved end extensions. Cofinal extensions deserve more attention than they have traditionally received.

1 The splitting theorem The fundamental theorem on cofinal extensions is Gaifman's Splitting Theorem, which not only establishes their existence but also reveals one of their most basic properties. Briefly, the Splitting Theorem asserts that every extension of nonstandard models splits into an *elementary* cofinal extension followed by an end extension. In particular, it follows that cofinal extensions are always elementary.

Unfortunately, the Splitting Theorem is language dependent. If we add a few new relations to the language of arithmetic, the theorem could well become false. For this reason, there are two additional versions of the theorem. The simplest form it takes, valid for any language (provided full induction is assumed), is the following:

1.1 Elementary Splitting Theorem *Let $\mathfrak{M} \prec \mathfrak{N}$ be models of arithmetic. There is another model $\mathfrak{M}^{cf}$ of arithmetic satisfying*

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