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Quick Completeness Proofs for Some Logics of Conditionals

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Introduction We start from the idea that a conditional $\alpha \rightarrow \beta$ is true iff α & $\sim\beta$ is either an impossibility or at least a *remoter* possibility, in some sense, than α & β. Let us try to make this precise.

First, we fix a *language* for the logic of conditionals: Let $\mathcal{L}$ be the set of formulas obtainable from the variables $p_1, p_2, p_3, \ldots$ using the arrow and the usual truth-functional connectives (viz., the true $\top$, the false $\bot$, negation $\sim$, conjunction &, inclusive disjunction v, material $\supset$ and $\equiv$). For $A \subseteq \mathcal{L}$ finite, $\bigwedge A$ denotes the conjunction, $\bigvee A$ the disjunction, of all elements of A (suitably grouped); e.g., $\bigwedge\phi = \top$, $\bigvee\phi = \bot$.

Second, we fix a notion of *model.* Let $\mathcal{M}$ be the set of all pairs (W, R), with W a nonempty set and R a trinary relation on it. For $x \in W$, we set $W_x = \{y\colon \exists z\, Rxyz\}$, and we require that R satisfy the following *reflexivity* and *transitivity* requirements:

$$\forall x \in W\, \forall y \in W_x\, Rxyy$$
$$\forall x \in W\, \forall y, z, w \in W_x\, (Rxyz \mathbin{\&} Rxzw \supset Rxyw).$$

A *model-class* is any $\mathcal{N} \subseteq \mathcal{M}$ closed under isomorphism; the interesting examples are obtained by imposing certain *characteristic restrictions* on R.

Next, we fix a notion of *satisfaction/validity.* A *valuation* in $(W, R) \in \mathcal{M}$ is a map V assigning each variable p_i a subset of W. V can be extended to all of $\mathcal{L}$ by treating truth-functions in the usual way (e.g., $V(\sim\alpha) = W - V(\alpha)$, $V(\alpha \mathbin{\&} \beta) = V(\alpha) \cap V(\beta)$), and defining $V(\alpha \rightarrow \beta)$ as the set of all $x \in W$ such that:

$$\forall y \in W_x \cap V(\alpha)\, \exists z \in W_x \cap V(\alpha)\, [Rxzy \mathbin{\&} \forall t \in W_x \cap V(\alpha)(Rxtz \supset t \in V(\beta))]$$

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