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General Models of Set Theory

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Quine explains the concept of set by considering an open sentence and observing that

> The notion of class is such that there is supposed to be, in addition to the various things of which that sentence is true, also a further thing which is the *class* having each of those things and no others as member. It is the class determined by the open sentence. ([4], p. 1)

The task of set theory is to formulate this idea rigorously in a way that blocks the paradoxes. In addressing the problem, technical investigations of set theory have been confined to what might be called its "internal structure", in which the discussion is framed in an underlying logic containing only those primitive predicates essential for formulating set theory itself. However, the intuitive notion of class is one in which sets can be specified by any clear extra-set theoretical condition. For example, informal asides in expositions of set theory frequently illustrate basic ideas using sets specified by open sentences such as "x is blue", "x is a man", "x is a parent of y", and so on. Further, the notion of an interpretation in model-theoretic semantics appears to require that sets in the domain of the model be specified by predicates of the object theory that are no part of the set theory in terms of which the model is formulated. Most importantly, however, if the notion of class is to be used in formalizing everyday or scientific discourse, for example, to explicate the application of mathematics to empirical subject matter, specification of sets by extra-set theoretical formulas is inescapable.*

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