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The Completeness of Intuitionistic Propositional Calculus for its Intended Interpretation

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If certain plausible though not absolutely compelling assumptions about choice sequences are admitted as postulates of intuitionistic analysis, then the usual formal system of intuitionistic propositional calculus can be proved complete for its intended interpretation: Any formula of the system which is intuitively correct no matter what propositions of intuitionistic mathematics are substituted for its variables can be formally deduced as a thesis of the system. This result has been known for some twenty years now, Kreisel's [5] being the first fully worked-out version. But thus far a streamlined and self-contained account of Kreisel's completeness theorem has been lacking in the literature. The aim of the present notes is to supply that lack.

We work with a well-known equivalent, presented in Section 1, of Heyting's 'classic' axiomatization [2]. The first step in a proof of completeness of such an axiomatization for its intended interpretation is always to prove completeness for some useful though artificial *un*intended interpretation, e.g., the topological models of McKinsey and Tarski, or the tree models of Beth.

We prefer to work with the relational models of Kripke, presenting in Section 2 a proof of Kripke's completeness theorem which, like the original proof [8] (so far as the latter pertains to propositional calculus), is finitistic. For a finitistic proof, ours is relatively quick and painless.

The Outlaw Schema, the choice-sequence assumption on which our work depends, is expounded in Section 3. Its statement requires only symbols for the basic operations of logic and arithmetic, and quantification over natural numbers and infinite sequences thereof. Our work requires (besides the Outlaw Schema) only noncontroversial axioms of intuitionistic logic and arithmetic.

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