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A Weak Free Logic with the Existence Sign

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In [1] I constructed a semantics for the system *FLI** of quantification and identity theory having

(A0) A, *where A is a tautology*
(A2) $\forall x(A \supset B) \supset (\forall xA \supset \forall xB)$
(A3) $A \supset \forall xA$
(A4) $a = a$
(A5) $a = b \supset (A \supset A^b//a)$
(A6) $\forall xA^x/a$, *where A is an axiom*

as axiom-schemata and

(R1) If A and $A \supset B$ are theorems, then B is a theorem

as rule of inference.[1] In the present paper, I want to extend this semantical analysis to the system *FLI*** which results from adding to *FLI** Leonard's weakened Principle of Specification

(A1″) $(\forall xA \ \&\ \mathrm{E}!a) \supset A^a/x$

(for which see [4]). Before doing this, however, I must spend a few words in motivating the enterprise.

As was pointed out in [1], in *FLI** the schema

(A1′) $(\forall xA \ \&\ \exists x(x = a)) \supset A^a/x$

is provable; hence, having in mind the definition of existence in terms of identity that free logicians have used at least since [3] and that we can express in the form

(1) $\mathrm{E}!a =_{df} \exists x(x = a)$, where x is the first individual variable in the alphabetical order,

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