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GENERALISED LOGIC II

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1 This paper is a continuation of [1] in which generalised sentential logic is fully developed in a sequence of axiom systems designated *GL0* to *GL5*. In Section 2 a minor adjustment is made to the system of [1] to form the system *GL0*, and *GL1* is then formed by adding an axiom implicit in the discussion in [1]; *G.L.0* and *G.L.1* are further variants. The next two sections break new ground by adding axioms to greatly strengthen sentential generalised logic: the resulting systems are *GL2*, *G.L.2*, and *GL3*, *G.L.3*. In Section 5 it is shown that *G.L.3* captures a conventional five-valued logic, *C.L.3*, based on truth tables and that a further five-valued logic, *C.L.5*, is characteristic of an arbitrary extension of *G.L.3* designated *G.L.5*. This result is used to prove consistency and further metatheorems about the earlier systems. In Section 6 the five-valued analysis is used for further developments pointing beyond the scope of the paper.

Theorems of some system, say *GLx*, are designated "xT. . ." (thus the theorems of [1] become 0T. . .). The designation "xT. . ." implies that I do not believe xT. . . is a theorem of a weaker system of the paper than *GLx*, but not that I have proved this. Metatheorems are designated "MT. . .", Heyting's sentential logic "HL", Boolean sentential logic "BL", and generalised logic (any system) "GL". Expressions of the form "*x*(. . *y* . .)" (e.g., "*N*(. .?. .)", "?(. .?. .)",) are used to designate kinds of formula within which there are occurrences of the monadic operator *y* dominated, in a subformula or the whole formula, by the monadic operator *x*.

2 The systems discussed in this section are *GL0*, *GL1*, *G.L.0*, and *G.L.1*, and the axioms discussed are:

ECfgEKfgf . . . . A19.   *E$\underline{N}$fEfKsNs* . . . . A20.

[1] includes the definition D1, *Cfg* = *EKfgf* and this blocks the full development of GL. The reason is that a definition sanctions interchangeability of the definiens and the definiendum in all contexts and so, for example, D1 gives *E?Cpq?EKpqp*, which by A22 and A24 (discussed in **3**) is not a thesis of GL. Arbitrary definitions are, of course, admissible, but D1 is not arbitrary because, for example, *C* is derivable from subordinate

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