

## N-POLAR LOGIC OF CLASSES

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**1** *Class calculus* In the following we make reference to certain notions without explaining them, but they have the usual meaning. Among these notions are: set, element, belonging, nonbelonging, decomposition, validity, etc.\*

### **1.1** *General definitions*

D1.1: A set  $M_1$  is a *subset* of the set  $M$ , if all elements of the set  $M_1$  are elements of  $M$ . If  $M_1$  is a subset of the set  $M$ , then  $M$  is an extended set (extension) of  $M_1$ .

D1.2: We call *bidisjunctive subsets* of the set  $M$  two subsets  $M_1$  and  $M_2$ , which have no common elements; that is to say, if an element belongs to the subset  $M_1$  this element does not belong to the subset  $M_2$ , while if an element belongs to the subset  $M_2$  that element does not belong to the subset  $M_1$ .

D1.3: We call *n-disjunctive subsets* of the set  $M$  the subsets  $M_i$ ,  $i = 1, 2, \dots, n$ , in a way that these subsets should be bidisjunctive in pairs.

D1.4: If a set can be decomposed following a certain criterion (intensional or extensional) into  $n$  *n-disjunctive* subsets, so that each element of the set  $M$  should be at the same time an element of one of these subsets, then these subsets are called *classes*. In order to differentiate between the classes and the other subsets, we shall indicate the classes with  $a_i$ ,  $i = 1, 2, \dots, n$ . The  $n$  classes will be listed arbitrarily and numbered  $a_1, a_2, \dots, a_n$ . Although the listing has been made arbitrarily, it will be maintained throughout the calculation. The deduced formulas for sets and subsets will be, of course, valuable for classes as well.

D1.5: If a set can be decomposed into  $n$  classes, we say that this set has *variance*  $n$ , and the classes of this set are *n-variant*. If to each element of

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\*Concerning the notion of polarity, see Léon Birnbaum, "Algèbre et logique tripolaire," *Notre Dame Journal of Formal Logic*, vol. XVII (1976), pp. 551-564.