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SIMPLIFYING THE AXIOMS OF THE PREDICATE CALCULUS

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1 *Introduction* It is often useful, e.g., in algebraic research, to have the postulates of a formal system expressed in the simplest possible form—"simple" meaning here: with a minimum of "metamathematical" (i.e., English) comments. The aim of the present paper* is to "simplify" the system of Quine [6], amended to allow the use of free variables.[1]

Only three "metamathematical" notions will be used: closure, bound substitution, and free substitution. They will be denoted by special symbols.

(i) $\mathcal{C}$ is the closure in [6] [e.g., the closure of $R(x, y)$ is $\forall x \forall y R(x, y)$].

(ii) $\mathcal{B}^x_y$ means: substitution of y for every bound occurrence of x [for instance $\mathcal{B}^x_y(P(x) \wedge \forall x R(x, y))$ is $P(x) \wedge \forall y P(y, y)$].

(iii) $\mathcal{F}^x_y$ means: substitution of y for every free occurrence of x [for instance $\mathcal{F}^x_y(P(x) \wedge \forall x R(x, y))$ is $P(y) \wedge \forall x R(x, y)$].

$\mathcal{B}^x_y A = A$ means that x is not bound in A; $\mathcal{F}^x_y A = A$ means that x is not free in A.[2]

2 *The proposed system* A, B, etc., will denote formulas; x, y, etc., will denote individual variables; $v_1, v_2, \ldots, v_n$ will denote distinct individual variables, the natural order of the indices showing the *alphabetic order* of the variables.

In a formula such as $\mathcal{C}A$, $\mathcal{C}$ denotes the string $\forall v_{i_1} \forall v_{i_2} \ldots$, where $v_{i_1}, v_{i_2}, \ldots$, are the variables which have at least one free occurrence in A, and with $i_1 < i_2 < \ldots$.

System I is:

(I1) $\vdash \mathcal{C}((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$
(I2) $\vdash \mathcal{C}(A \Rightarrow (B \Rightarrow A))$
(I3) $\vdash \mathcal{C}((\neg A \Rightarrow \neg B) \Rightarrow (B \Rightarrow A))$

*This paper is chiefly the development of an abstract already published (see [4]).

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