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CALCULEMUS

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In this paper I shall develop three methods of expressing propositions in algebraic notation along with purely computational tests for validity. The first two methods pertain to syllogistic arguments, the third to propositional logic. All seem to be of theoretical as well as pedagogical interest.

1 The Additive Method Traditional schematic formats for representing syllogisms have no doubt suggested some elementary mathematical operations to numerous logicians, but full mathematization has proved elusive. Fred Sommers [6], to my knowledge, was the first person to devise an adequate system whereby syllogisms can be treated as additions. It turns out that one can add up the premisses of a valid syllogism and the sum will be the uniquely correct conclusion. Sommers' system is, however, encumbered with several nonmathematical rules; and we lack a convincing explanation as to why we should expect his method to work. In reflecting on these issues, I was able not only to uncover how an additive system works and why, but also to produce a greatly simplified method.

The efficacy of my method depends on being able to capture mathematically certain traits of the syllogism and especially a complete set of three rules for determining validity employed by Wesley Salmon [4]. A syllogism has exactly two premisses and a conclusion, all of subject-predicate form. Furthermore, the three propositions always involve exactly three terms (which may be either subjects or predicates), one of which (the middle term) appears once in each premiss but never in the conclusion. Thus, in adding premisses it is necessary that the middle term drop out; otherwise it would appear in the sum representing the conclusion. One of Salmon's rules states that the middle term must be "distributed" (see [3] for a discussion of this concept) exactly once. This calls for some provision for mathematically differentiating a distributed occurrence of the middle term in one premiss from its mandated, undistributed occurrence in the other premiss. One way to accomplish this is to have the middle term be positive once and negative once.

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