

A MODEL-THEORETIC EXPLICATION OF THE THESES OF KUHN AND WHORF

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1 Introduction We wish in this paper* to give a mathematical explication of the ideas usually associated with the names of Thomas S. Kuhn and Benjamin L. Whorf. These ideas concern the incommensurability of scientific theories and the effect of language on thought. We also touch on some related notions and applications. Hopefully our model-theoretic formulation will also have some interest for logicians and set theorists.

2 Preliminaries We consider several languages and the models associated with them; however, we want to characterize the models of our languages independently of any particular one of them. By taking all our models to be models of a certain set theory (we take Zermelo-Frankel set theory, **ZF**, for the sake of definiteness) and by interpreting the non-logical constants and relations of our languages to be fixed elements of the universe, the class of models in which a sentence of a language is true can be considered to be simply a class of models of **ZF**.

To be more precise we need the following definition:

Definition 1 A language $\mathcal{L}$ is of the form $K \cup \{\varepsilon\}$ where $K = \{c_j, R_j, f_j, Q_j\}$ is a finite collection of constant symbols, relation symbols, function symbols, and sort symbols, and where ε is a distinguished binary relation symbol. Sentences in $\mathcal{L}$ are built in the usual inductive way.

We also deal with languages of the form $\mathcal{L} = K_1 \cup K_2 \cup \{\varepsilon\}$ where the K_i are different formal languages. The intention is to think of the languages K_i as formal scientific languages. The symbol ε is added so that statements in any K_i as well as extra-linguistic observations can be described in some neutral formal language. Although we assume the universe to be set-theoretic, we do not assume that a K_i -scientist thinks in terms of sets, but

*Some of these ideas were first formalized by Randall, from whose work [4] parts of this paper are adapted.