Notre Dame Journal of Formal Logic Volume XXI, Number 1, January 1980 NDJFAM

## A REALIST SEMANTICS FOR COCCHIARELLA'S T\*

## JOHN CARSON SIMMS

**0** *Introduction* Russell's paradox has two versions. The first version concerns "the set of all sets which are not members of themselves". The second version concerns "the property of being a property which is not a property of itself": the so-called *Russell property*. This second version of Russell's paradox is called *Russell's paradox of predication*.<sup>1</sup>

Nino Cocchiarella\* designed a logistic system, which he christened  $T^*$ , whose purpose was to represent the original ontological context behind Russell's paradox of predication [10]. The grammar of  $T^*$  is essentially that of standard second-order logic but goes beyond it by allowing predicate terms to occupy subject positions in the formulas of  $T^*$ . Cocchiarella generated the axioms and inference rules of  $T^*$  by explicitly and appropriately generalizing the axioms and inference rules of Church's formulation of standard second-order logic [1] to the extended grammatical context of  $T^*$  and by adding a new axiom schema whose effect is to represent the realist assumption implicit in the ontological background of Russell's paradox of predication that every relation<sup>2</sup> is an individual.

It is a remarkable fact that  $T^*$  is consistent. It even turns out that  $T^*$  is a conservative extension of standard second-order logic. Thus, Russell's "paradox" of predication is not really a paradox after all-at least not in the logistic context of  $T^*$ . These discoveries signify the genesis of a radically new, important, and fruitful approach to predication theory.<sup>3</sup>

Be forewarned, however, that  $T^*$  is not without its (apparent) ontological oddities. One particularly interesting example of such an oddity concerns identity. Cocchiarella [10] showed that indiscernibility cannot be construed as identity in  $T^*$  since, in the ontology of  $T^*$ , there must be properties which are indiscernible and yet not co-extensive. Going further, Meyer [14] showed that there is no binary relation in the ontology of  $T^*$ 

Many other nonstandard second- (and even higher-) order theories of predication have grown up around  $T^*$ , and we will call these logistic

<sup>\*</sup>My thanks to Professor Cocchiarella who made many valuable criticisms of an earlier version of this paper. And for introducing me to Logic.