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THE Ω-SYSTEM AND THE Ł-SYSTEM OF MODAL LOGIC

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1 *Definition* The Ω-system is a logistic system, the alphabet of which consists of a denumerable set of propositional variables ($p_1, p_2, \ldots$), and of three connectives: $\Rightarrow$ (implication), $\neg$ (negation), and Ω. Ω is a 0-ary connective, i.e., a propositional constant. The well-formed formulas are defined as usual[1] throughout this paper,* the letters "x", "y", and "z" represent arbitrary wffs. We have three axiom schemas and a rule:

$$x \Rightarrow (y \Rightarrow x)$$
$$(x \Rightarrow (y \Rightarrow z)) \Rightarrow ((x \Rightarrow y) \Rightarrow (x \Rightarrow z))$$
$$(\neg x \Rightarrow \neg y) \Rightarrow (y \Rightarrow x)$$
$$\frac{x, x \Rightarrow y}{y}$$

The system is different from the well-known Frege-Łukasiewicz system for the classical propositional calculus, for the wffs are not the same: they may contain the symbol Ω, but Ω does not appear explicitly in the postulates.

The Ω-system may be considered as a modal system when possibility P and necessity N are defined as follows:

$$Px = \Omega \Rightarrow x$$
$$Nx = \neg P \neg x$$

whence $Nx = \Omega \wedge x$.

The chief result of this paper is that this modal system is (in a certain sense) identical with the Ł-system of modal logic. The Ł-system is defined in Łukasiewicz [13] (see Harrop [7] or Rose [23], if that paper is not available); see also Łukasiewicz [12], Anderson [1], Smiley [24], Church [4].

*This paper is the development of a communication given to the "Colloquium on non-classical logics" (Helsinki, 1962). Only a mimeographed abstract had been circulated.

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