Notre Dame Journal of Formal Logic Volume XX, Number 4, October 1979 NDJFAM

THE ALGEBRA OF RELATIVES

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The term "algebra of relatives" is sometimes regarded as being no more than an outmoded name for the algebra of relations. At best it is associated with the methods and notation developed by C. S. Peirce and E. Schröder. It is true that in Peirce's later work, which was taken up by Schröder, a "relative" is regarded as a set of ordered pairs, hence as a relation. Yet there is evidence that in Peirce's early work he thought of a relative as being something entirely different; a collection of individuals, each of which is related in a given way to some other individual. In the course of its development the algebra of relatives thus metamorphosed into an algebra of relations. The aim of this paper is to see how much sense can be made of an algebra of relatives, where "relative" is taken in its Peircean sense. I first give a sketch of the historical background, then an informal account of relatives and their behaviour, and finally consider how this might be given a sound set-theoretical basis. It turns out that the informal "algebra" of relatives holds this title only by courtesy; nevertheless, by means of a certain artifice, one can give a set-theoretical imitation of it which is an algebra in the strict sense of the word.

1 In Chapter 7 of the *Categories* Aristotle introduces the notion of a relative as follows ([1], 6a36):

We call relatives all such things as are said to be just what they are, of or than other things, or in some way in relation to something else.

The first impression one gets from this passage is that certain individuals are called relatives, namely those that are always referred to in relation to something else, but this impression is countered by the following remark in the translator's Notes:

He [Aristotle] does not say that 'larger' and 'slave' are relatives, but that the larger and the slave are relatives. However, he does not, of course, mean that, for example, the slave Callias is a relative (he is a substance), but that Callias is a relative in so far as he is *called a slave*; in other words, 'slave' is a relative term.

This passage points out the fundamental difficulty in the conception of a