

## MODAL INTERPRETATIONS OF THREE VALUED LOGICS. I

MICHAEL J. DUFFY

**0 Introduction** The present paper\* extends a result of Peter Woodruff's reported in [1], to the effect that the three-valued logic  $\mathbf{L}$  of Łukasiewicz may be interpreted as a modal system. Woodruff obtains his result by constructing a mapping from the wffs of  $\mathbf{L}$  to those of the modal system  $S5$ . A definition is then produced which gives directions for the construction of interpretations of  $\mathbf{L}$  from interpretations of  $S5$ , and it is further shown that no interpretation of  $\mathbf{L}$  fails to be thus obtainable. The result is of interest especially because it has been argued that  $\mathbf{L}$  cannot plausibly be viewed as a modal system, even though Łukasiewicz himself viewed it as one.<sup>1</sup> Here the question of the existence of modal interpretations of  $\mathbf{L}$  via other mappings into  $S5$  is explored. In order that the present paper be self-contained, no familiarity with [1] is presupposed; but the reader familiar with that work will appreciate this author's indebtedness to it.

In what follows, we use ' $p$ 's and ' $q$ 's as syntactic variables for wffs of both  $\mathbf{L}$  and  $S5$ , trusting the context to signal which system is under discussion. It will be convenient to use the bracketless Polish notation, presumed to be familiar.

**1 The Systems  $\mathbf{L}$  and  $S5^2$**  We suppose  $\mathbf{L}$  to be constructed from a denumerably infinite set of atoms, the set of wffs then being the least set that both contains the atoms and has  $Cpq$  and  $Np$  as members whenever  $p$  and  $q$  are members. An interpretation  $I$  for  $\mathbf{L}$  is any function from the set of wffs to  $\{1, \frac{1}{2}, 0\}$  such that  $I(Np) = 1 - I(p)$  and  $I(Cpq) = \min(1, 1 - (I(p) - I(q)))$ . A wff  $p$  of  $\mathbf{L}$  is valid (contravalid) if, for every  $I$ ,  $I(p) = 1(0)$ ; otherwise  $p$

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