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A SEMANTICAL ACCOUNT OF THE VICIOUS CIRCLE PRINCIPLE

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Russell claimed that statements about *all* propositions are meaningless (Russell [4], p. 63 and Russell and Whitehead [5], p. 37). Here we attempt to give formal expression of Russell's view by developing a semantical account of propositional quantification that has the vicious circle principle as a consequence. According to the account we give the vicious circle principle bears an interesting relation to the view that there are no extraordinary sets (sets which are members of themselves).

1 We focus on a particular formal system, **F**, which contains these symbols: **I** (a binary connective to be read 'The proposition that . . . is the same as the proposition that ______'), Π (the universal quantifier), sentence variables ($p_0, p_1, p_2, \ldots$), sentence constants ($q_0, q_1, q_2 \ldots$). **F** contains as formulas p_n, q_k, $\mathbf{I}\phi\psi$ and nonvacuous quantifications $\Pi p_n\psi(p_n)$. A sentence is a formula with no free occurrences of variables.

Philosophical considerations count against interpreting Π substitutionally. Consider, for example, that all instances of 'Some sentence of English says that p' are true, but its universal quantification seems false. It seems false because for any language there are more truths and falsehoods (e.g., about numbers) than can be expressed in that language. So a model of **F** has to be so defined that, for some model $\mathfrak{M}$, $\Pi p_n\psi(p_n)$ is false in $\mathfrak{M}$ while $\psi(\alpha)$ is true in $\mathfrak{M}$ for every admissable substituend α of the variable p_n. Such a model must consist in part of a set of propositions such that each of the α's get assigned a proposition but some of the propositions get assigned to no α.

Next a semantical account of **F** must provide a way of evaluating identities

$$\mathbf{I}q_0q_1 \;,$$
$$\mathbf{I}q_0\Pi p_0p_0 \;,$$
$$\mathbf{I}\Pi p_0p_0\Pi p_1p_1 \;,$$

and so on. One way of dealing with this is to structure the domain so that it

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