

ŁUKASIEWICZ'S TWIN POSSIBILITY FUNCTORS

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Jan Łukasiewicz, in his paper "A system of Modal logic",¹ introduces what he calls "the twin possibilities",² viz. the 'Δ' and '∇' functors. The existence of these two functors is, he claims, something of a logical paradox, for they are identical when defined apart, i.e., as parts of separate matrices, but non-identical when defined together, i.e., as parts of one matrix. It is my purpose in this paper to point out that Łukasiewicz's arguments in this matter are faulty, and that the apparent paradox dissolves in the light of a reasonable criterion for determining identity of matrices. Łukasiewicz creates his \mathfrak{M}_9 matrix, i.e.,

	<i>C</i>	(5,7)	(5,8)	(6,7)	(6,8)	<i>N</i>	Δ
\mathfrak{M}_9	* (5,7)	(5,7)	(5,8)	(6,7)	(6,8)	(6,8)	(5,7)
	(5,8)	(5,7)	(5,7)	(6,7)	(6,7)	(6,7)	(5,7)
	(6,7)	(5,7)	(5,8)	(5,7)	(5,8)	(5,8)	(6,7)
	(6,8)	(5,7)	(5,7)	(5,7)	(5,7)	(5,7)	(6,7)

by multiplying the matrices

	<i>C</i>	5	6	<i>N</i>		<i>C</i>	7	8	<i>N</i>
\mathfrak{M}_7	*5	5	6	6	\mathfrak{M}_8	*7	7	8	8
	6	5	5	5		8	7	7	7

together using the equalities

- (a) $C(a,x)(b,y) = (Cab,Cxy)$
- (b) $N(a,x) = (Na,Nx)$
- (c) $\Delta(a,x) = (a,Cxx)$

where 'a' and 'b' represent elements of \mathfrak{M}_7 and 'x' and 'y' represent elements of \mathfrak{M}_8 . He abbreviates the \mathfrak{M}_9 matrix by allowing '1' to stand for

1. *The Journal of Computing Systems*, vol. 1 (1953), pp. 111-149.
 2. *Ibid.*, p. 127.