

## A FORMAL INTERPRETATION OF ŁUKASIEWICZ' LOGICS

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One way of coming to understand the meaning of the concepts in a non-standard logic is by seeing how that logic can be interpreted within some more familiar, more readily understood, logic. A case in point is intuitionism. Given the intuitionistic account of the nature of mathematics, it is natural to expect that, from a classical point of view, intuitionistic logic is concerned with the assertability or provability of sentences, and not merely with their truth. Here interpretation can sharpen insight. For, as McKinsey and Tarski [1] show, intuitionistic propositional logic is exactly interpretable in the modal system S4 under a number of very natural mappings.

In the present note, I shall use the method of interpretation, and indeed one of the McKinsey-Tarski mappings, as a way of sharpening some recent views about the meaning of Łukasiewicz' many-valued logics. Several logicians have contended that these systems can plausibly be regarded as logics of exactness. The various values represent degrees of truth, with the top value being complete truth, the bottom value complete falsity, and the intermediate values degrees of partial truth. If this idea stands scrutiny, then Łukasiewicz' logics may be appropriate tools to use in developing logics for vague terms. (For discussion, see [2] and [3]).

The following model suggests itself as a formal representation of the view just mentioned. Suppose  $n$  judges are asked to decide if a certain claim (e.g., "Gerald Ford is bald.") is true or false. Suppose the only responses permissible are true and false and that the judges are a fair cross-section of the population. Then the verdict of the judges can be represented by an  $n$ -place sequence of  $t$ 's and  $f$ 's. How can we model the degree to which this claim is true? Given that the judges are a fair sample, a reasonable view is that the degree of truth is shown by the number of judges who say that the claim is true. And this number  $k$  is naturally represented by the  $n$ -place sequence of truth-values which begins with  $k$   $t$ 's and has  $f$ 's from then on. So, I propose that the degree of truth  $d(A)$  of  $A$  be construed as the sequence  $\langle t, t, \dots, t, f, \dots f \rangle$  where the sequence associated with  $A$  contains exactly  $k$   $t$ 's.

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