Notre Dame Journal of Formal Logic
Volume XX, Number 2, April 1979
NDJFAM

ADMISSIBLE SETS AND RECURSIVE EQUIVALENCE TYPES

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Recently there has been much interest in admissible sets. Part of this is due to the fact that the constructive notions of finiteness and recursion can be extended to include infinite sets and operations. In such a structure, recursive equivalence types can be defined which correspond to the classical ones. We shall show that the Cantor-Bernstein Theorem and the Tarski Cancellation Law hold in a straightforward manner. However, a satisfactory definition of an isol depends upon the admissible set. We shall exclude projectible admissible sets which have elements that include large Σ_1 definable subsets. Also, we shall need a weak uniformizing procedure to tie together recursive enumerability and Σ_1 definability. With these conditions some of the equivalences that hold for isols can be extended to admissible sets. We shall conclude with a stronger definition of an isol which preserves a cancellation law similar to that for the ordinary isols.*

1 *Definitions and propositions* The following definition and Propositions 1-6 are due to Jensen [3]. The definition will give great flexibility in defining functions. The proofs of the propositions are elementary and can be found in [1]. Throughout we shall consider a non-empty transitive set $\mathfrak{M}$. Our language contains the predicates = and ϵ with their usual interpretation and constant symbols for each $x \epsilon \mathfrak{M}$; we shall use the same symbol for both object and name. We allow *bounded quantification* $\forall x \epsilon y\varphi$ and $\exists x \epsilon y\varphi$. Those formulas which contain only bounded quantifiers are called Σ_0 *predicates*. They are closed under the operations $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$, and $\forall x \epsilon y$, $\exists x \epsilon y$. We are particularly interested in the $\Sigma_1(\Pi_1)$ *predicates* of the form $\exists x\varphi(\forall x\varphi)$ where φ is Σ_0. We say $\mathfrak{M}$ is *admissible* if $\mathfrak{M}$ satisfies the following axioms (called **PZF**):

(1) Axioms of the empty set, pairing, and union.

*Most of the material in this paper appears in the author's Ph.D. thesis (Rutgers University, 1972), supervised by Professor Erik Ellentuck whose help and interest were indispensible.

Received April 2, 1975