Notre Dame Journal of Formal Logic
Volume XX, Number 1, January 1979
NDJFAM

ALTERNATIVE AXIOMATIZATIONS OF ELEMENTARY PROBABILITY THEORY

WILLIAM K. GOOSENS

1 *Outline* In this paper I offer some new axiomatizations of elementary (finite) probability theory. The new axiomatizations have the following distinctive features. First, both conditional and unconditional probabilities are simultaneously axiomatized. Second, the axiomatizations are extensions of Kolmogorov probability theory, and have the usual definitions linking conditional and unconditional probabilities as theorems. Third, while both probabilities are simultaneously axiomatized, the resultant axioms are about as simple as the usual axiomatizations of conditional probability alone. Fourth, the axioms adopted have strong and direct justifications independently of the interpretation of the probability function (axiomatization **L** however is only a permissible extension of intuitions). The reader who wishes to review the axiomatizations before their rationale can at this point go to section **5** (the main axiomatization is system **KK**).

2 *What is unsatisfactory about usual axiomatizations?* I offer alternative axiomatizations because I am dissatisfied with the customary ones. But this dissatisfaction has a special nature. In improving existing axiomatizations, one might reduce the number of axioms, or simplify them, or adopt axioms that make proofs easier and more elegant. Although in my own axiomatizations I am concerned with these factors, none of these traditional formal motivations is the source of my dissatisfaction. Instead, I am basically concerned about how the relationship between conditional and unconditional probabilities is developed (for some related misgivings, see de Finetti [2], pp. 81-83). There are two probability functions about which we have intuitions. The basic formal difference is that unconditional probability is a one-place function, and conditional probability is a two-place function. As a result, there are two probabilistic concepts that can be axiomatized, and the two can be axiomatized independently. What makes a function a one-place probability function, and what makes a function a two-place probability function, are separate questions, and the correct answer for each involves a separate characterizing theory.

Received March 16, 1975