Notre Dame Journal of Formal Logic
Volume XX, Number 1, January 1979
NDJFAM

THE EXTENDED CALCULUS OF INDICATIONS INTERPRETED AS A THREE-VALUED LOGIC

FRANCISCO J. VARELA

1 *Introduction* The point of view of indication, as a foundational notion for mathematical thinking, was introduced by G. Spencer Brown in 1969 [1]. Taking as primitive only the intuitive notion of distinction or indication, he presented a simple yet amazingly powerful calculus, the Calculus of Indications (**CI**), whose full import is slowly being recognized [2]. In its two values, indicated, '⌐', and not indicated, ' ', this calculus embodies the general form of any two-valued situation. Many possible interpretations of **CI** are thus possible, but a particularly interesting one is for classical propositional logic, where statements can be true or false (*cf.* [1], Appendix 2). I have taken the Calculus of Indications as a starting point in an attempt to produce adequate tools to deal with self-referential situations.* Self-reference is, of course, of great historical importance; it was responsible for a major crisis in mathematical thinking at the turn of the century. More recently, with the development of cybernetics and systems theory, other aspects of self-referential situations have become apparent, namely, the fact that many highly relevant systems have a self-referential organization. The key character of self-production in living systems is, perhaps, the most obvious instance; examples from the neurological, cognitive, and social domains also abound [3,4,5,6]. With his motivation I developed an Extended Calculus of Indications (**ECI**), capable of dealing with the basic forms of self-reference, and thus, providing a foundation to interpret any possible instance of them [7]. The point of view of indication greatly simplifies the discussion of self-referential situations, by simply having an expression indicate itself. Expressions where self-indication is allowed, are called boolean expressions of higher degree by Spencer Brown, and in his [1] he hinted at their possible applications. In [7] I showed that **CI** is not consistent with self-indicating expressions and derived **ECI**, where not two but three values exist: indicated, not indicated, and self-referring or

*This work was supported by National Science Foundation Grant #GB-36922.

Received June 7, 1975