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DOMAIN RESTRICTIONS IN STANDARD DEDUCTIVE LOGIC

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Introduction Although this paper draws upon logical evidence associated with Gödel-type undecidability arguments, its concern is independent of current work in the philosophy of mathematics. The issue to be examined is the conceptualization of a system of deductive logic as applicable to a domain, both infinite and comprehensive, of individual classes or sets of numbers. Historically, the notion of an infinite domain was introduced into deductive logic for the purpose of accommodating number theory and thus establishing a logical foundation for the conventions of mathematical practice. Yet subsequent developments in mathematical logic have made it evident that a host of formal difficulties attend any naive understanding of the infinity hypothesis. Moreover, it is no longer certain that the notion of a comprehensive ontological domain for classical logic is a primary conceptual need. The focus of attention has shifted from the importance of standard logic for mathematical theory to the impact of mathematical analysis upon our understanding of what constitutes a logical system. Questions of logical domain seem less relevant to mathematics proper than to issues involving the nature and ontological status of a formalized language.

In this paper the evidence both of the class contradiction and of Gödel's undecidability proof will be used as evidence against the assumption of an ontologically total domain for either a typed postulational logic or an untyped logic with the class membership predicate as primitive. The notion most essential to my argument concerns the status of individual or predicative constants when logical schemata are supplemented by actual class or property names. If we say that a logical calculus has the capacity to represent formally an infinite number of classes or properties, we are committed to the view that the formalized language in some sense contains an infinite supply of interpreted constant symbols which can serve as replacements for variables. However, any formula in which such constants are employed will contain only symbols that are discoverable within some finite list of the expressions of the language. It is this characteristic of a

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