Notre Dame Journal of Formal Logic
Volume XX, Number 1, January 1979
NDJFAM

ON UNSOLVABILITY IN SUBRECURSIVE CLASSES OF PREDICATES

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Introduction Decision problems for various classes of sets or languages have been studied extensively in the theory of automata and formal languages [7]. These results have been combined in [3] where certain closure properties important to formal language theory are used to show undecidability in classes meeting certain specifications, and in [6] where the "ability to count" provides the unsolvability. In this paper* closure properties prevalent in mathematical logic will be utilized to relate several decision problems to each other. First, several decision problems for bases of the r.e. sets will be located in the arithmetic hierarchy and then other decision problems will be related to them.

1 *Preliminaries* In the results presented below $\mathcal{R}_*$ will always be a class of recursive predicates [never the class of *all* recursive predicates] for which there is a recursive enumeration [denoted: $R_0, R_1, \ldots$]. The class of predicates $\mathcal{Q}_*$ will always be a subclass of $\mathcal{R}_*$. In addition, the class $\mathcal{R}_*$ will always possess an s_n^m Theorem, which can be stated:

Theorem (s_n^m) *There is a recursive function $s_n^m(i, x_1, \ldots, x_m)$ such that for any R_i in $\mathcal{R}_*$:*

$$R_{s_n^m(i,x_1,\ldots,x_m)}(y_1, \ldots, y_n) \equiv R_i(x_1, \ldots, x_m, y_1, \ldots, y_n).$$

This technical theorem will be useful in proofs since it provides a method to set several variables to specific values.

Several notational conventions should be given at this point. Script letters refer to classes of predicates while capital Roman letters represent predicates and small letters denote functions and variables. The empty set is denoted by $\emptyset$ and $\mathbf{N}$ represents the set of non-negative integers. The sequence $W_0, W_1, \ldots$ is the "standard enumeration" of the recursively

*This research has been supported in part by the National Science Foundation under Grants GJ-30076 and MCS 77-06321.

Received March 20, 1975