Notre Dame Journal of Formal Logic
Volume XX, Number 1, January 1979
NDJFAM

THEORY OF OBJECTS AND SET THEORY: INTRODUCTION AND SEMANTICS

ANDRÉ CHAUVIN

1 *Preliminary* By Kleene's *Introduction to Metamathematics* (1952), CH. XIV, §72, Theorems 35 and 38, *if consistent*, a first order theory has "*Kleene's models*", i.e., models where the universe is the set of natural integers 0, 1, . . ., n, . . . and the fundamental relations are each of the form $P(a_1, \ldots, a_n)$ with

$$P(a_1, \ldots, a_n) \leftrightarrow (\exists x)(\forall y) R(a_1, \ldots, a_n, x, y) \leftrightarrow (\forall x)(\exists y) S(a_1, \ldots, a_n, x, y)$$

where R and S are primitive recursive and "$\leftrightarrow$" is the biconditional.

Trying to prove the consistency of first order *set theory* by finding directly such a model (which must exist if this theory is consistent), we thought, as partial recursive functions are basic notions of Theory of Algorithms, to which belong Kleene's models, that it would be better to bring *set theory* nearer to *theory of algorithms*, by taking for the first one as *primitive* the notion of *partial* function and the relation of equality and as *derived* the notion of set and the relation of membership (if we do not impose extensionality to our partial functions we are again nearer to the primitive basic notion of the theory of algorithms, say the notion of algorithm itself).

We were confirmed in this idea by reading Von Neumann's "Eine Axiomatizierung der Mengenlehre". As a matter of fact, Von Neumann takes as primitive the notion of (total) function and the relation of equality. But he stresses that his system presents some arbitrariness. *We thought that this arbitrariness lay at the very beginning of Von Neumann's system, when he divides a priori its objects ("Ding") between* I-*objects* x (arguments) *and* II-*objects* f (functions), in such a way that the domain O of Von Neumann's objects is the union of O_I, the domain of arguments, and O_{II}, the domain of functions, and when he considers that the result of the binary operation [,] of application of a *function* to an argument, say $[f, x]$, is defined on and only on the cartesian rectangle $O_{II} \times O_I$. Von Neumann admits again (implicitly) that the term $[f, x]$ has always a meaning, f being always taken in O_{II} and x in O_I, and he states that this term describes an argument (a I-object).

Received January 2, 1974