Notre Dame Journal of Formal Logic
Volume XX, Number 1, January 1979
NDJFAM

RECURSIVE EQUIVALENCE TYPES ON RECURSIVE MANIFOLDS

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*Preliminaries** Standard recursive theory is worked on $N = \{0, 1, 2, \ldots\}$. In this paper the theory is worked on recursive manifolds. An *enumeration* of a set A is a map from N onto A. If the enumeration is injective, it is said to be an *indexing*. The ordered pair $\langle A, \mathfrak{A}\rangle$ is said to be a *recursively enumerable manifold* (**REM**, for short) if A is the union of enumerated sets (i.e., for some index set P, there is a collection of enumerations $\{\alpha_p\}_{p \epsilon P}$ with $A = \bigcup_{p \epsilon P} A_p$, where $A_p = \alpha_p(N)$), with the enumerations satisfying certain conditions. $\mathfrak{A} = \{\alpha_p\}_{p \epsilon P}$ is called the *atlas*. Each A_p is called a *patch*. For a set $S \subseteq A$ each $\alpha_p^{-1}(S)$ is called a *pullback* (into N) of S. To make $\langle A, \mathfrak{A}\rangle$ an **REM**, we require that each $\alpha_q^{-1}(A_p)$ must be recursively enumerable (r.e., for short) and the domain of a partial recursive function (p.r. function, for short) f into $\alpha_p^{-1}(A_q)$ satisfying $\alpha_q = \alpha_p \circ f$. If each $\alpha_q^{-1}(A_p)$ is recursive (rec., for short), the manifold is a *recursive manifold* (**RM**). If each α_p is an indexing, $\langle A, \mathfrak{A}\rangle$ is an *injective* **REM** (**IREM**). A manifold which is both an **RM** and an **IREM** is an *injective recursive manifold* (**IRM**). A manifold such that each patch nontrivially intersects at most finitely many other patches is said to be *finitary*. For reasons that will appear in the proofs of the first two theorems, all manifolds considered in this paper will be assumed to be finitary **IRM**'s unless otherwise specified. $\langle N, \mathsf{I}\rangle$ is defined to be the finitary **IRM** with $\mathsf{I} = \{\alpha\}$, $\alpha(n) = n$.

If $\langle A, \mathfrak{A}\rangle$ is an **REM** and $\langle B, \mathfrak{B}\rangle$ is another **REM** with enumerations β_q, $q \epsilon Q$, the cartesian product $A \times B$ can be given a manifold structure as follows: for each $\langle p, q\rangle$ in $P \times Q$, let $A_p \times B_q$ be a patch of $A \times B$ enumerated by $\gamma_{p,q}$ where $\gamma_{p,q}(\sigma(n, m)) = \langle \alpha_p(n), \beta_q(m)\rangle$, σ being the standard rec. bijection from N^2 onto N. This manifold on $A \times B$ is called the *direct product* of

*This paper is a thesis written under the direction of Professor Vladeta Vučković and submitted to the Graduate School of the University of Notre Dame in partial fulfillment of the requirements for the Degree of Doctor of Philosophy with Mathematics as the major subject in August 1976. The author is indebted to his director, Professor Vladeta Vučković, for his patient and constant assistance.

Received July 15, 1976