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Conjunctive Normal Forms and Weak Modal Logics Without the Axiom of Necessity

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In *S5* it is known that any formula can be reduced to conjunctive normal form (CNF) of degree 1; completeness easily follows from this fact (see, e.g., [5]). The purpose of this paper is to extend this method to prove completeness for very weak modal logics, and to give some applications. Twenty modal logics are dealt with here. We call them Lpq, $DLpq$ ($p = 0, 1$ and $q = 0, 2$), $L3$, $DL3$, and $\mathbf{L}_N$, where $\mathbf{L}$ is one of $L00$–$DL3$. We first define $\mathbf{L}$-tautologies corresponding to each logic $\mathbf{L}$, where $\mathbf{L}$ is either Lpq or $DLpq$, and characterize them by the set $\mathbf{L}^*$ of value-assignments having certain properties. Then, we show that $\Diamond A \vee \Box B_1 \vee \ldots \vee \Box B_n \vee C$ is provable in a modal logic $\mathbf{L}$ iff (1) C is a tautology, or (2) $A \vee B_i$ is an $\mathbf{L}$-tautology for some i, where C, but not necessarily A or B_i, contains no modal operator. Completeness for Lpq (or $DLpq$) follows from this equivalence. For completeness of $\mathbf{L}_N$, where $\mathbf{L}$ is either Lpq or $DLpq$, we shall also make use of the above equivalence for $\mathbf{L}$. For the remaining logics, a more direct method will be used.

The modal logics dealt with in this paper are defined in Section 1, $\mathbf{L}$-tautology definitions and characterizations are given in Section 2, completeness proofs in Section 3, and applications in Section 4.

1 Weak modal logics We are given a countable set, Π, of propositional variables, logical connectives, $\sim$, $\wedge$, $\vee$, $\rightarrow$, $\Box$, and parentheses, (,). The other connectives and formulas are defined as usual. We shall consider modal systems obtained by adding the following axiom schemata and rules of inference to the classical logical base.

A1 $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$
A2 $\Box A \rightarrow \Diamond A$

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