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A Pair of Nonisomorphic $\equiv_{\infty\lambda}$ Models of Power λ for λ Singular with $\lambda^{\omega} = \lambda$

SAHARON SHELAH*

Let $L_{\infty\lambda}$ denote the infinitary logic that is formed by allowing arbitrary conjunctions of formulas and either existential or universal quantifications over sets of fewer than λ variables. The formulas are taken to be those having fewer than λ free variables. It is easy to see that any structure of cardinality less than λ is characterized up to isomorphism by a sentence of $L_{\infty\lambda}$. Scott [5] showed that if $M \equiv_{\infty\omega} N$ and $\|M\| = \|N\| = \aleph_0$, then $M \cong N$, where $\equiv_{\infty\omega}$ denotes elementary equivalence in $L_{\infty\omega}$. Chang [1] generalized this by showing that if cf $\lambda = \omega$ then $M \equiv_{\infty\lambda} N$, $\|M\| = \|N\| = \lambda$, still implies $M \cong N$. However, Morley, gave an early unpublished example which showed that for any regular $\lambda > \omega$, there are structures M,N such that $\|M\| = \|N\| = \lambda$, $M \equiv_{\infty\lambda} N$, but $M \ncong N$. His structures were trees of height λ, one with a branch of length λ and the other without. The reader may wish to consult Dickmann [2], Nadel [3], Nadel and Stavi [4], or Stavi [13] for more work in this direction.

The above results leave open the situation for singular λ, with cf $\lambda > \omega$. The purpose of this paper is to show that if $\lambda^{\omega} = \lambda$, then there are structures M,N with $\|M\| = \|N\| = \lambda$ such that $M \equiv_{\infty\lambda} N$ but $M \ncong N$. Under the GCH, cf $\lambda > \omega$, $\lambda > \omega$, implies $\lambda^{\omega} = \lambda$, so, in this situation, the entire picture would be known.

The reader may also be interested in consulting Shelah [8], [9], [11], and [12], which deal with the question of the cardinality of $\{N/\cong : N \equiv_{\infty\omega} M, \|M\| = \|N\| = \lambda\}$, or Shelah [6], Theorem 1, [7], Chapter XIII §1, and [10], which are concerned with finding models of particular theories. In [12] our main result is proved for many more λ of cofinality $>\aleph_0$ (but for different structures).

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