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First-Order Theories as Many-Sorted Algebras

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Introduction In this paper, by developing a study of first-order logic through many-sorted algebras, we show that every first-order theory is a particular algebra verifying axioms in equational form (see Section 2); therefore we are able to apply Birkhoff's theorems concerning the varieties (see Section 1 and [7] and [8]) and to obtain the Henkin models algebraically, whence the completeness theorem of first-order logic (see Section 3).

The many-sorted (or heterogeneous) algebras, systematized by Birkhoff and Lipson (see [4] and [10]), find a natural application in investigating programming languages: several approaches to the formal definition of semantics for such languages can be developed by means of morphisms between many-sorted algebras (see [2] and [6]).

This paper shows the analogous possibility of algebraizing linguistic features of logic, thus yielding a unique framework for the formal analysis of both programming and logical languages within universal algebra.

The analysis here developed is strongly related to those of [1], [2], [11], and [12]. Moreover, an approach to the topic of present paper is elaborated in the monographs [8] and [9]. Knowledge of these works is not necessary to understand what follows, though it would better enable one to appreciate our results.

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