Notre Dame Journal of Formal Logic
Volume 25, Number 1, January 1984

Vector Spaces and Binary Quantifiers

MICHAŁ KRYNICKI, ALISTAIR LACHLAN
and JOUKO VÄÄNÄNEN

1 ***Introduction*** Caicedo [1] and others [3] have observed that monadic quantifiers cannot count the number of classes of an equivalence relation. This implies the existence of a binary quantifier which is not definable by monadic quantifiers. The purpose of this paper is to show that binary quantifiers cannot count the dimension of a vector space. Thus we have an example of a ternary quantifier which is not definable by binary quantifiers.

The general form of a binary quantifier is

$$Qx_1y_1 \ldots x_ny_n\phi_1(x_1,y_1) \ldots \phi_n(x_n,y_n).$$

An example of such a quantifier is (in addition to all monadic quantifiers) the similarity quantifier:

$$Sx_1y_1x_2y_2\phi_1(x_1,y_1)\phi_2(x_2,y_2) \longleftrightarrow \phi_1(\cdot,\cdot) \text{ and } \phi_2(\cdot,\cdot)$$
$$\text{are isomorphic as binary relations.}$$

We let $\mathcal{L}(Q)$ denote the extension of first-order logic by the quantifier Q. Recall the definition of $\Delta(\mathcal{L}(Q))$ from [2]. It is proved in [4] that $\Delta(\mathcal{L}(S))$ is equivalent to second-order logic. Even monadic quantifiers can have very powerful Δ-extensions. Thus, simple syntax (such as $\mathcal{L}(Q)$) is no guarantee for simple model theory.

2 ***Vector spaces—the main lemma*** Let K be an infinite field. We shall consider vector spaces

$$\mathcal{V} = \langle V, +, \cdot, 0; K\rangle$$

over K. Here + denotes addition of vectors, $\cdot$ denotes multiplication of vectors by an element of the field, and 0 is the zero vector. Thus $\mathcal{V}$ should be considered as a two-sorted structure. Let L denote the language associated with $\mathcal{V}$

Received April 15, 1982