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Decomposable Collections of Sets

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The notion of a *weave* was first defined by Gaisi Takeuti as an approach to the problem of Borel Determinateness. In a paper co-authored by Burd and Takeuti [1] some of the game-like properties of weaves were explored. A weave is a set-theoretic object that corresponds to a two-person game in which each player presents a *choice* of moves from a set of possibilities rather than a single move. In another paper [2] Green and Takeuti used the weave idea to prove a theorem about Boolean polynomials.

The Green-Takeuti paper gives sufficient conditions enabling a Boolean polynomial to be factored into a Boolean statement in which no atom appears more than once. Consider a Boolean polynomial to be a collection of sets, each atom being an element, each term being a set of elements. In this context the Green-Takeuti theorem is a theorem about the decomposition of collections of sets.

In this paper we present a new proof of the Green-Takeuti theorem, and extend the proof to cover the case where the collection of sets is infinite (i.e., the Boolean polynomial is infinitary). We do this using the notion of a weave.

Definition 1 Let P be a set. Let $\mathbf{W}$ and $\mathbf{W}'$ be nonempty subsets of $\mathcal{P}(P) - \{\phi\}$. The pair $\langle\mathbf{W},\mathbf{W}'\rangle$ is a Weave of P iff both:

(1) for each set W in $\mathbf{W}$, and each set W' in $\mathbf{W}'$, the intersection $W \cap W'$ is a singleton set
(2) for each element p in P, there is a set W in $\mathbf{W}$ and a set W' in $\mathbf{W}'$ such that $W \cap W' = \{p\}$.

The set P is called the set of Points of the weave. We denote this by writing $P = \text{Points}(\langle\mathbf{W},\mathbf{W}'\rangle)$. Notice that Clause 2 in the definition of a weave

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