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On the Freyd Cover of a Topos

IEKE MOERDIJK*

A theory is said to have the disjunction-property (*DP*) if whenever a disjunction $\phi \vee \psi$ is provable in the theory, either ϕ or ψ must be provable. As is well-known, many theories for intuitionistic arithmetic and analysis have the *DP*. The *DP* for intuitionistic type theory was first established by Friedman. More recently, a purely topos theoretic proof has been given by Freyd. An extensive discussion of both methods can be found in [4]. Although Freyd's construction is much more elegant, A. Ščedrov and P. Scott have shown that the two methods are essentially the same in [7].

A question that arises immediately is the following: If one adds new symbols and a particular set of axioms T to the logical axioms and rules, does the resulting higher-order theory still have the *DP*? Some instances of this question in which T consists of a single axiom have been considered in [5]. In this note, we will obtain a syntactic description of a class of theories that have the *DP* by investigating some of the logical properties of the Freyd cover, thus extending the results of [5].

The results will *not* cover many of the higher-order analogues of theories of intuitionistic arithmetic and analysis which are known to have the *DP*. One reason for this is that, from a more logical point of view, the Freyd cover lacks many nice properties. For an alternative type of cover that fills this gap, the reader is referred to [6].

In the first section of this paper, we will motivate the Freyd cover from a more logical perspective. There is probably nothing new in this, but it still is important to realize that what is really going on is a straightforward generalization of more traditional methods used in the model theory of first-order

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