

Halldén-Completeness by Gluing of Kripke Frames

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1 Introduction We give in this paper a sufficient condition, cast in semantic terms, for Halldén-completeness in normal modal logics, a modal logic being said to be *Halldén-complete* (or ‘Halldén-reasonable’) just in case for any disjunctive formula provable in the logic, where the disjuncts have no propositional variables in common, one or other of those disjuncts is provable in the logic.

It is clear from this definition that the weakest normal modal logic, the system K , is not Halldén-complete, in view of the provability in K of the formula $\Box \perp \vee \Diamond \top$, since neither disjunct is a theorem of K and we may rewrite $\perp$ as $p \wedge \sim p$ and $\top$ as $q \vee \sim q$. (Notation and terminology are as in [5]; some historical and philosophical remarks on Halldén-completeness may be found in [3].) Thus the Halldén-complete normal logics are either extensions of $K + \Box \perp$ (the ‘Absurd’ system) or of $K + \Diamond \top$ (the system D). Amongst the latter systems are such familiar modal logics as T , $S4$, and $S5$, already known to be Halldén-complete. There are several proofs in the literature for these systems and other isolated cases of systems in this spectrum, though these have an *ad hoc* appearance about them in the sense that they tend to exploit rather specific properties of the individual systems concerned (see, e.g., [4]). We point to a common semantic principle which brings some order into the situation, and see how far it takes us in general. It turns out (Theorem 2, below) that a very simple semantic condition is sufficient for Halldén-completeness amongst the extensions of D .

2 P-morphic fusion The main theorem of the paper requires first a definition.