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An Axiomatization of Predicate Functor Logic

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1 Introduction Predicate Functor Logic is a formal system devised by Quine to provide a natural variable-free equivalent of elementary logic. It is described, in its most recent form, in [9]; but the ideas go back to [8] and [7]. In [9] Quine discusses the problem of exhibiting a simple and complete "proof procedure" for predicate functor logic, i.e., a procedure for recursively enumerating the formulas of predicate functor logic whose elementary logic counterparts are valid. This paper describes one such procedure.[1] We provide an interpretation for predicate functor logic which is consonant with Quine's remarks. The class of formulas valid with respect to this interpretation is axiomatized by a recursive set of axioms and rules. The proof that this axiomatization is complete is an adaptation of the Henkin completeness proof for elementary logic. It does not require translations between predicate functor and elementary logic. The axiomatization is not as simple as might be desired: nonprimitive symbols are needed to display the axioms conveniently. But a closely related system is shown to have a simple and perspicuous axiomatization.

2 Predicate functor logic The language of predicate functor logic contains symbols of two varieties. First, for each $n \geqslant 0$ there is a countable collection of *n-ary atomic predicates*. For convenience we take these to be just the n-ary predicates of elementary logic. Second, there are the *predicate functors*, $\urcorner$, $\cap$, $\mathbf{p}$, $\mathbf{P}$, [,], and I.

For $n \geqslant 0$ the set of *n-ary predicates* satisfies the following conditions:

1. All n-ary atomic predicates are n-ary predicates.
2. If P^n and Q^m are n-ary and m-ary predicates, respectively, then $(P^n \cap Q^m)$ is a $max(m,n)$-ary predicate.

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