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Minimally Incomplete Sets of Łukasiewiczian Truth Functions

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By an *n-valued truth function* we shall understand a function on the set $\{1,2,\ldots,n\}$. If such a function is closed on the set $\{1,n\}$, it will be said to be *pure*. And, if it can be defined by composition from $\neg$ and $\rightarrow$, it will be referred to as *Łukasiewiczian*. Here:

$$\neg p = (n - p) + 1$$

and

$$(p \rightarrow q) = \max[1,(q - p) + 1].$$

In [3] it is proved, for the three-valued case, that the set of Łukasiewiczian functions and the set of pure functions are one and the same. It is also observed, again in the three-valued case, that if f is non-Łukasiewiczian, then $\{\neg, \rightarrow, f\}$ is functionally complete (i.e., all three-valued functions can be defined by composition from $\neg$, $\rightarrow$, and f). The import of the latter result is that although $\neg$ and $\rightarrow$ are together functionally incomplete, their incompleteness is *minimal*. That is, when $\{\neg,\rightarrow\}$ is supplemented with a "new" function, the resulting set is always functionally complete.

It is the purpose of the present essay to establish a more general result from which the previous two can be derived as corollaries:

Theorem 3 *The following are equivalent if* $2 < n$:

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