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On the Borel Classification of the Isomorphism Class of a Countable Model

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Introduction For ρ, a countable similarity type, let X_ρ be the space of structures of similarity type ρ whose universe is ω (see [13], Section 3). For any element $\mathcal{A}$ of X_ρ, let $[\mathcal{A}]$ be the set of all elements of X_ρ which are isomorphic to $\mathcal{A}$. Scott [10] showed that $[\mathcal{A}]$ is a Borel subset of X_ρ. In fact, he showed that for any such $\mathcal{A}$ there is a sentence θ of $L_{\omega_1\omega}$ such that $[\mathcal{A}]$ is exactly the set of elements of X_ρ which are models of θ (see [1], Ch. VII, for a good write-up of Scott sentences).

In [13] Vaught considerably strengthened Scott's result. There is a natural hierarchy of formulas of $L_{\omega_1\omega}$. Let $\Pi_0^0 = \Sigma_0^0$ be the quantifier-free first-order formulas. For any $\alpha \geqslant 1$ the Π_α^0 formulas are those of the form:

$$\bigwedge_{n<\omega} \forall x_1 \forall x_2 \ldots \forall x_n \theta_n$$

where each θ_n is $\Sigma_{\beta_n}^0$ for some $\beta_n < \alpha$. The Σ_α^0 formulas are those of the form:

$$\bigvee_{n<\omega} \exists x_1 \exists x_2 \ldots \exists x_n \theta_n$$

where each θ_n is $\Pi_{\beta_n}^0$ for some $\beta_n < \alpha$. A set $B \subseteq X_\rho$ is called invariant iff it is closed under isomorphism. Vaught showed that for every Π_α^0 invariant set B there is a Π_α^0 sentence θ such that B is the set of models of θ, and similarly for Σ_α^0.

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