Frege's Definition of Number

STEVEN WAGNER*

I will interpret Frege on the main issues about his definition of numbers as extensions at [8], §68: its point, its correctness, and its implications for the nature of number. My view, in short, is that Frege understood this definition (henceforth *D*) as a partly free construction of the numbers. His escape from the subjectivism this seems to entail was to relegate differences between equally correct constructions to a Kantian realm of appearance.

As background for the problems I want to discuss, let me note a few of the relatively clear facts about *D* (cf. [1]). Frege's interest in defining number is subordinate to a dominant goal of proving the arithmetical laws. Many of these, of course, had already been established in number theory, but in Frege's view the axioms that number theorists simply assume should be proved as well (§§1-4).[1] In fact, two relatively independent points of view motivate this demand. For the mathematician it is simply a matter of proving whatever can be proved. Even if the axioms are entirely certain, their proof will advance mathematics by revealing mathematically interesting connections between propositions. But these connections also bear on philosophical questions about the analyticity or apriority of arithmetic. On Frege's understanding of analyticity, for example, analyticity will be shown only by a derivation of the axioms from purely logical laws. A search for such derivations is therefore the