

## A Note on Initial Segment Constructions in Recursively Saturated Models of Arithmetic

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This note is something of an addendum to our earlier paper [5], in which, with the aid of a simple method of constructing elementary initial segments of recursively saturated models of arithmetic, we constructed a continuum of elementarily inequivalent structures  $(\mathfrak{N}; \mathfrak{M})$ , where  $\mathfrak{N}$  was a fixed countable recursively saturated model of  $PA$  and  $\mathfrak{M}$  a recursively saturated elementary initial segment of  $\mathfrak{N}$ . Herein we take a close look at this method, recounting past glories, offering new facts, and citing a few minor open problems.

The basic construction we are referring to was originally performed in joint work with Jonathan Stavi. In Section 1, we review this construction and the original application from [7], and follow it up with a few related observations. Section 2 reviews the application cited above and, again, follows it up with a few related observations. This application produces a large variety of pairs,  $(\mathfrak{N}; \mathfrak{M})$ , with  $\mathfrak{M}$  a recursively saturated elementary initial segment of a given countable recursively saturated model  $\mathfrak{N}$ . In Section 3, we ask the question of obtaining variety without the basic construction. This question has two senses: First, can we construct a large number of decidedly distinct such pairs,  $(\mathfrak{N}; \mathfrak{M})$ , without using the basic construction? And second, can we construct such a large number of such pairs,  $(\mathfrak{N}; \mathfrak{M})$ , where  $\mathfrak{M}$  is not of the form provided by the basic construction? We have only partial, but positive, solutions to

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