574
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Definability in Self-Referential Systems

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Introduction Self-referential systems are theories formulated in a (typed) first-order language $\mathcal{L}$, in whose intended interpretation the predicates refer to objects which are themselves predicates of $\mathcal{L}$. In order to avoid the Russell–Zermelo paradox, Hiller and Zimbarg Sobrinho [1] introduced *self-referential systems with involution*: these are theories whose intended models admit an elementary embedding into itself, denoted by '*', and called *involution map*. As a consequence, the universe of predicates inherits a structural hierarchy of objects, classified into countably many *types*.

A peculiar property of types refers to the size of the universe of their corresponding objects: the larger a type, the smaller its domain, and for this reason, types have been suggestively taken as negative integers: 0, -1, -2, and so on.

The main properties satisfied by self-referential systems with involution were outlined in [2], and are the following:

(a) $\mathcal{L}$ possesses unrestricted (or universal) variables
(b) all predicates are extensional
(c) the Comprehension axiom for starred formulas is true
(d) (Definability condition) every element in the universe of a realization is definable by a one-free-variable formula of $\mathcal{L}$.

The first three clauses above can be directly expressed in our (typed) first-order language $\mathcal{L}$ without any further ado. With respect to the Definability condition, however, it is not altogether clear how it could be formulated in a first-order version of self-reference, due to its obvious higher-order character. The purpose of this article is to present first-order axioms which, added to $\mathcal{W}$ (see [1]), produce the same effect as the apparently stronger 'Definability condition'.

1 Hiller's problem Realizations of self-reference in which the Definability condition holds have been referred to as *intended models*. It is well-known that

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