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Infima of Recursively Enumerable Truth Table Degrees

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Introduction Let r be a reducibility between sets of natural numbers. It is natural to study structural properties of the partial order of r-degrees, and for varying values of r such studies have formed one of the main thrusts of classical recursion theory. Because of the importance of recursively enumerable (r.e.) sets, particular attention has focused on the study of the r.e. r-degrees (i.e., those r-degrees which contain an r.e. set). For most (but not all) values of r, both the r-degrees as a whole and the r.e. r-degrees form upper semi-lattices but not lattices; that is, every pair of r-degrees in the structure has a supremum, but some pairs do not have an infimum (inf). For this reason, much attention has been paid to constructing sets whose r-degrees have some specified behavior with respect to the inf(imum) operation.

The basic internal problem is the existence of substructures with specified infima, i.e., the lattice embedding question. For some reducibilities this problem has been essentially solved. Thus, for example, the sublattices of the r.e. m- and wtt-degrees are exactly the countable distributive ones by Lachlan ([7] and [8]) and Stob ([15]). On the other hand, every recursively presented lattice is embeddable in the r.e. tt-degrees by Fejer and Shore ([4]). The situation for the r.e. T-degrees is, however, quite complex. All countable distributive lattices are embeddable ([10] and [16]) as are some ([8]) but not all ([9]) finite nondistributive ones. The general problem for T-degrees remains open. Ambos-Spies and Lerman ([1]) present the current state of affairs.

Another basic question concerns the way the r.e. r-degrees sit inside all the r-degrees. In particular, one would want to know what is the relationship, if any, between the inf of $\mathbf{a}_0$ and $\mathbf{a}_1$ considered as elements of the structure of the r.e. r-degrees and the inf of $\mathbf{a}_0$ and $\mathbf{a}_1$ considered as elements of the structure of all r-degrees. A priori, there are five possibilities if $\mathbf{a}_0,\mathbf{a}_1$ are r.e. r-degrees:

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