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Pointwise Definable Substructures of Models of Peano Arithmetic

ROMAN MURAWSKI*

Let PA be Peano arithmetic formalized in a first-order language $L(PA)$ with $0, S, +, \cdot$ as nonlogical symbols and based on the usual Peano axioms with the axiom scheme of induction. Let M be a model of PA. Since we have in PA definable Skolem functions, $Def(M) < M$ where $Def(M)$ is the substructure of M with the universe consisting of elements definable in M without parameters. If M is a nonstandard model, then we have in M nonstandard formulas. Therefore we can consider substructures of M analogous to $Def(M)$ with universes consisting of points definable by certain nonstandard formulas and initial segments of M generated by such pointwise definable substructures.

After recalling some basic information on satisfaction classes we give the precise definition of pointwise definable substructures. We distinguish two cases: (a) definability without parameters bigger than the defining formulas and (b) definability with a parameter bigger than the defining formulas. We consider properties of such substructures and of their families.

1 Introduction A serious approach to the possibility of nonabsoluteness of the finite (and so of the logical syntax too) was realized first by Robinson in [15] where he has also shown that nonstandard languages have no uniquely determined semantics. Krajewski (in [11]) has explicitly introduced and has studied the notion of a satisfaction class.

Recall that if M is a nonstandard model of PA and $F\dot{m}$ is a formula of $L(PA)$ strongly representing in PA the recursive set of Gödel numbers of formulas of $L(PA)$ (cf., e.g., [1] and [16]) then we have in M nonstandard objects a such that $M \models F\dot{m}[a]$. We call them nonstandard formulas. They determine a nonstandard language which we denote by $Form(M)$. To speak about its

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