

Operational Semantics for Positive R

I. L. HUMBERSTONE*

1 Two kinds of formal semantics for intensional logics It is convenient to begin with a few remarks about the distinction between model-theoretic (sometimes called 'set-theoretic') semantics and algebraic semantics for sentential logics containing non-truth-functional connectives. Both issue in definitions of validity on a structure or over a class of structures in terms of which completeness theorems are sought, to the effect that provability in this or that logic coincides with a certain such notion of validity. I take the hallmark of the model-theoretic approach to be that it characterizes the validity notion in question via an inductively defined notion of truth of a formula at a point in a model, while the algebraic approach features no such intermediate level of description.¹ The considerable appeal of the Kripke relational semantics for normal (and some non-normal) modal logics and for intuitionistic and intermediate logics over earlier algebraic accounts was due no doubt in part to its supplying this intermediate level of description, with something recognizably analogous to the informal notion of truth restored to center stage. This feature of the relational semantics for modal logic is shared by the operational semantics suggested for certain normal systems by Garson ([9]), as well as by the neighborhood semantics for these (and weaker) systems. Accordingly, in such cases, even when the structures are algebraic structures (carrier set + operation(s)) what we have is model-theoretic rather than algebraic semantics.

The above distinction is somewhat stipulatively drawn, articulating just one significant difference often marked by the terminological contrast. It certainly ignores, in particular, a tendency on the part of some writers to speak of any proposed formal semantics as 'merely algebraic' as opposed to 'genuine' semantics when they are not persuaded that it throws any light on the intended meanings of the expressions involved (see [4] for example).

*In addition to those mentioned in the text and in the notes of this paper, I should like to thank this Journal's referee for corrections to an earlier draft, and my former student Michaelis Michael for helpful conversions on some of the topics touched on.