

Wittgenstein versus Turing on the Nature of Church's Thesis

S. G. SHANKER

I 'Turing's machines are humans who calculate' The title of this paper suggests two highly contentious claims: first, that Wittgenstein was aware of the developments in recursion theory that took place during the 1930s, and second, that he disputed the version of Church's Thesis (hereafter CT) which Turing had presented in 'On Computable Numbers' [36]. It will be best to concede at the outset that both themes represent something of a critical liberty; or rather, a corollary. For the subject of this paper is really Wittgenstein's attack on the mechanist terms in which Turing had interpreted his computability results. But one of the central points that Turing was to make in his 1947 'Lecture to the London Mathematical Society' was that the Mechanist Thesis is not just licensed but is in fact *entailed* by his 1936 development of CT [39]. Wittgenstein's argument thus demands careful scrutiny of both the relation of Turing's argument to CT and the cognitivist implications that have been read into CT as a result of Turing's influence.

Before we consider these matters, however, we must first satisfy ourselves that Wittgenstein was indeed intent on repudiating Turing's computability thesis. For it has long been a source of frustration to Wittgenstein scholars that no overt mention of this issue is found in *Lectures on the Foundations of Mathematics: Cambridge 1939* [48]. Indeed, until recently it might have been thought that the title of this paper makes the still further unwarranted assumption that Wittgenstein was even aware of 'On Computable Numbers'. Any such doubts were laid to rest by the discovery of an off-print of the essay in Wittgenstein's *Nachlass*, and even more important, the following mystifying reference to Turing machines occurs in *Remarks on the Philosophy of Psychology*:

Turing's 'Machines'. These machines are *humans* who calculate. And one might express what he says also in the form of *games*. And the interesting games would be such as brought one *via* certain rules to nonsensical instructions. I am thinking of games like the "racing game". One has received the

Received May 12, 1987