520
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Turing Projectability

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1 Introduction Let N be the set of natural numbers. A function $g: N \to N$ is *effectively* (or *mechanically*) computable if there is an algorithm or effective procedure which, given (a representation of) an integer n as input, terminates after finitely many steps, and yields (a representation of) $g(n)$ as output. The Church–Turing thesis (CT) states that the effectively computable number-theoretic functions are precisely the Turing computable number-theoretic functions, or, equivalently, the recursive functions. In this paper, we develop a generalization of the notion of mechanical computability and use CT to argue for an analogous thesis concerning that generalization.[1]

It is an important aspect of the classical notion of effective or mechanical procedure that an effective procedure terminate for each natural number given as input.[2] We consider here a generalization of effective computability according to which a *nonterminating* effective process may be said to determine the values of a number-theoretic function. Our idea is to describe such a process in the following terms. There is, first, an effective procedure (in the ordinary sense) which, applied to any input m, generates a computation that proceeds in stages $\sigma_{1m}, \sigma_{2m}, \ldots$. Secondly, there is an effective function f which, applied to any stage σ_{nm} of the computation, yields a *projection*, or tentative value, for the computation at the stage n. The projection is subject to revision, in the sense that at a further stage σ_{km} $(k > n)$, it may happen that $f(\sigma_{km}) \neq f(\sigma_{nm})$. But we allow the projected value to change only *finitely* often, so that for each m there is a value p such that

$$\exists n \forall k > n\, f(\sigma_{km}) = p.$$

We say that the given pair of effective procedures is a *projective strategy*. The function g is said to be *projected* by that strategy if and only if for each m an n can be found such that

$$\forall k > n\, f(\sigma_{km}) = g(m).$$

In this case, we say that the strategy is *stable* for m at stage n.[3]

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