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Eventual Periodicity and "One-Dimensional" Queries

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Abstract We expand on the automata-like behavior of monadic second order relations investigated by Buchi and Ladner. We present a generalization of their representation theorem and use it to separate the intersection of the classes of monadic existential second order and monadic universal second order queries from the class of one-dimensional inductive queries.

0 Introduction In this article, we compare monadic second order logic to monadic least fixed point logic.

The first notion applies to second order sentences: the dimension of a second order sentence is the maximum arity of its quantified relation variables. Most of the work on this notion of dimension is restricted to one-dimensional ("monadic") second order relations, starting with the automata-like behavior of monadic second order queries described in Buchi [5] and Ladner [17]. These papers extended Ehrenfeucht's [9] pebble game-theoretic characterization of the Fraisse [12] equivalence relation for first order relations to monadic second order relations.

We will use this game in order to compare monadic second order logic with a slightly smaller logic whose relation with monadic second order logic is significant. Recall that on ordered structures, by Fagin [10], existential second order logic corresponds to NPTIME, whereas by Immerman [14] and Vardi [22] PTIME corresponds to a logic called Least Fixed Point (LFP) by computer scientists (see Aho & Ullman [3], Chandra & Harel [6], and Immerman [15]) and Positive Elementary Induction by logicians (see Moschovakis [20]).

One of the deepest problems in logic and theoretical computer science is the relationship between LFP and second order logic. First of all, LFP $\subseteq \Pi^1_1$. On $\mathfrak{N} = \langle \omega, +, \times, \uparrow, 0 \rangle$, where "$\uparrow$" refers to exponentiation, LFP $= \Pi^1_1$ (Kleene [16]). On the other hand, over any class of finite structures, LFP is closed under negation (see [15]) and hence over such a class LFP $\subseteq \Delta^1_1$. Since a number of que-

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