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Unary Interpretability Logic

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Abstract Let T be an arithmetical theory. We introduce a unary modal operator '**I**' to be interpreted arithmetically as the unary interpretability predicate over T. We present complete axiomatizations of the (unary) interpretability principles underlying two important classes of theories. We also prove some basic modal results about these new axiomatizations.

1 Introduction The language $\mathcal{L}(\Box)$ of propositional modal logic consists of a countable set of proposition letters $p_0, p_1, \ldots$, and connectives $\neg$, $\wedge$, and $\Box$. $\mathcal{L}(\Box, \rhd)$ is the langauge of (binary) interpretability logic, and extends $\mathcal{L}(\Box)$ with a binary operator '$\rhd$'. ('$A \rhd B$' is read: 'A interprets B'.) The *provability logic L* is propositional logic plus the axiom schemas $\Box(A \to B) \to (\Box A \to \Box B)$, $\Box A \to \Box\Box A$, and $\Box(\Box A \to A) \to \Box A$, and the rules Modus Ponens ($\vdash A, \vdash A \to B \Rightarrow \vdash B$) and Necessitation ($\vdash A \Rightarrow \vdash \Box A$). The *binary interpretability logic IL* is obtained from L by adding the axioms

($J1$) $\Box(A \to B) \to A \rhd B$
($J2$) $(A \rhd B) \wedge (B \rhd C) \to (A \rhd C)$
($J3$) $(A \rhd C) \wedge (B \rhd C) \to (A \vee B) \rhd C$
($J4$) $A \rhd B \to (\Diamond A \to \Diamond B)$
($J5$) $\Diamond A \rhd A$,

where $\Diamond \equiv \neg\Box\neg$. *IL* is taken as the base system; extensions of *IL* with one or more of the following schemas have also been studied:

(F) $A \rhd \Diamond A \to \Box\neg A$
(W) $A \rhd B \to A \rhd (B \wedge \Box\neg A)$
(M_0) $A \rhd B \to (\Diamond A \wedge \Box C) \rhd (B \wedge \Box C)$
(P) $A \rhd B \to \Box(A \rhd B)$
(M) $A \rhd B \to (A \wedge \Box C) \rhd (B \wedge \Box C)$.

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