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Pure Second-Order Logic

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Abstract Pure second-order predicate calculus is a predicate calculus where the only variables are predicate variables. In it, logical truth is decidable, and semantic consequence is compact. Pure second-order functional calculus is a functional calculus where the only variables are function variables. In it, semantic consequence is not compact, and there is no complete proof procedure for logical truth.

The language of the pure second-order predicate calculus consists of those formulas of the second-order predicate calculus whose only variables are predicate variables. A statement of its semantics will help to elucidate my notational conventions.

A model $\mathfrak{M}$ is a pair $\langle \mathfrak{D}, \mathfrak{R} \rangle$, where $\mathfrak{D}$ is a domain of individuals, and $\mathfrak{R}$ is a function such that:

1. for any name n, $\mathfrak{R}(n) \in \mathfrak{D}$;
2. for any k-adic function sign f, $\mathfrak{R}(f)$ is a k-adic operation on $\mathfrak{D}$;
3. for any k-adic predicate F, $\mathfrak{R}(F) \subseteq \mathfrak{D}^k$.

Let an S be a function such that for each k-adic predicate variable ϕ, $S(\phi) \subseteq \mathfrak{D}^k$. Let $S\langle s/\phi \rangle$ be just like S, save that $S\langle s/\phi \rangle$ assigns s, a subset of $\mathfrak{D}^k$, to ϕ. For each S, let $S[\ ]$ be such that:

1. for any name n, $S[n] = \mathfrak{R}(n)$;
2. for any function sign f, $S[f(t_1, \ldots, t_k)] = \mathfrak{R}(f)(S[t_1], \ldots, S[t_k])$;
3. for any predicate F, $S[F] = \mathfrak{R}(F)$;
4. for any predicate variable ϕ, $S[\phi] = S(\phi)$.

We say that in $\mathfrak{M}$ S satisfies:

1. an atomic formula $\mathfrak{f} t_1, \ldots, t_k$ iff $\langle S[t_1], \ldots, S[t_k] \rangle \in S[\mathfrak{f}]$;

2.1. $\neg A$ iff S does not satisfy A;

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