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An Alternative Rule of Disjunction in Modal Logic

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Abstract Lemmon and Scott introduced the notion of a modal system's providing the rule of disjunction. No consistent normal extension of KB provides this rule. An alternative rule is defined, which KDB, KTB, and other systems are shown to provide, while K and other systems provide the Lemmon-Scott rule but not the alternative rule. If S provides the alternative rule then either $\sim A$ is a theorem of S or A is whenever $A \to \Box A$ is a theorem; the converse fails. It is suggested that systems with this property are appropriate for handling sorites paradoxes, where $\Box$ is read as 'clearly'. The S4 axiom fails in such systems.

Lemmon and Scott introduced the notion of a modal system's *providing the rule of disjunction* ([6], p. 44). This paper investigates similar rules for systems that do not provide the rule of disjunction. It ends with an application to philosophical issues about vagueness and sorites paradoxes.

First, some definitions. For any wff A, $\Box^0 A = A$; $\Box^{j+1}A = \Box^j \Box A$. S is a modal system.

S provides the Lemmon-Scott rule of disjunction:
if $\vdash_S \Box A_1 \vee \ldots \vee \Box A_n$
then $\vdash_S A_i$ for some i $(1 \leq i \leq n)$.

S provides the weak rule of disjunction:
if $\vdash_S \Box^{j_1} A_1 \vee \ldots \vee \Box^{j_n} A_n$ for all $j_1, \ldots, j_n$ (≥ 0)
then $\vdash_S A_i$ for some i $(1 \leq i \leq n)$.

S provides the bad rule of disjunction:
if $\vdash_S A_0 \vee \Box A_1 \vee \ldots \vee \Box A_n$
then $\vdash_S A_i$ for some i $(0 \leq i \leq n)$.

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