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On Π_1^0 Classes and their Ranked Points

ROD DOWNEY

Abstract We answer a question of Cenzer and Smith by constructing a nonzero degree each of whose members is a rank one point of a Π_1^0 class. The technique of proof is a rather unusual full approximation argument. This method would seem to have other applications.

1 Introduction In this paper, we solve a problem of Cenzer and Smith [3] concerning ranked points for Π_1^0 classes in 2^ω. Here the reader should recall that a member (point) of a Π_1^0 class P is called *ranked* if, for some ordinal α, $x \notin D^\alpha(P)$ where D^α denotes the α-th Cantor-Bendixon derivative. This is defined via

$$D(P) = \{x \in P : x \in \mathrm{Cl}(P - \{x\})\}$$

$$D^0(P) = P,\ D^{\alpha+1}(P) = D(D^\alpha(P)), \text{ and}$$

$$D^\alpha(P) = \bigcap_{\beta<\alpha} D^\beta(P) \text{ for } \alpha \text{ a limit ordinal.}$$

The *rank* of a ranked point y *in* P is the least α with $y \notin D^{\alpha+1}(P)$, and the rank of y is the minimum rank in all P such that y is a ranked point in P. Ranked points in Π_1^0 classes have been extensively investigated in, for example, Jockusch and Soare [6,7], Clote [4], Cenzer et al. [1,2], and Cenzer and Smith [3]. It is known that all Δ_2 degrees contain ranked points [3], but not all points can be ranked. For example, all the nontrivial iterated jumps of $\varnothing$ cannot be ranked and all hyperimmune degrees (see Section 2) contain unranked points.

These results left the following question open: do all degree $\neq 0$ contain unranked points? In Section 2 we answer this question by showing that:

Theorem *There is a completely ranked degree below* $0''$, *that is, there exists* **a** *with* $0 < \mathbf{a} < 0''$ *such that if* $A \in \mathbf{a}$ *then* A *is ranked. Indeed for all* $B \leq_T A$, B *has rank* ≤ 1.*

*The author thanks Richard Shore for pointing out that this stronger conclusion follows from the proof in §2.

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