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A Simple Axiomatizable Theory of Truth

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Abstract A general framework for studying self-referential languages is given: by this is meant both a model theory and a complete set of axioms for that model theory. The generality of the approach is shown by exhibiting the wide range of pathological sentences it allows, and its model-theoretic compatibility with other approaches, such as semi-inductive ones. Philosophical motivation for some of the new technical moves is given, and an appendix supplies the completeness proof.

1 The most salient feature of the truth predicate is its apparent obedience to Criterion *T*. Criterion *T*, however, is so simple that one might have hoped that a predicate axiomatically duplicating this property of truth could be easily added to first-order logic much as equality has been. The Liar's Paradox dashes any thought that such a move is technically easy.

In any case, axiomatic treatments of the truth predicate have been largely ignored as a possibility in the standard literature.[1] One reason for thinking such approaches are ruled out is that it seems that a language cannot have a theory of truth without some capacity to describe its own syntax. But even a fairly weak syntactic capacity can breed incompleteness, and therefore (it seems) a theory of truth cannot be axiomatized either.

This problem can be solved by relativizing the syntactic capacity of the language to the model rather than fixing it once and for all for a class of interpreted languages. By this I mean, e.g., that instead of fixing a canonical mapping of constants to sentences (via a quotation function, say) that is to hold for all models of the language, one allows the constants to vary in what they are mapped to from model to model. Doing so results in a theory of truth that is valuable in two ways. First, it has been known since Tarski that syntactic theory and truth theory do not sit comfortably together. But it is the details—exactly how much syntax can sit together with exactly how much truth—that are of interest. There are some results (see Gupta [5], pp. 183–194 for some examples), but an approach that gives a general framework to study this question exhaustively is desirable. Such a framework should be one that contains a first-order theory of

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